Show

$$\int_1^\infty x^n e^{-x}\, dx$$

converges for any n without using IBP n times.

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Let $f(x) = x^n$ and $g(x) = e^x$.

$$\lim_{x\to\infty} \frac{f(x)}{g(x)}$$

$$= \lim_{x\to\infty} \frac{x^n}{e^x}$$

Using L'Hospital's rule n times,

$$\lim_{x\to\infty} \frac{x^n}{e^x} = \lim_{x\to\infty} \frac{n!}{e^x}$$

$$= 0$$

$\therefore\ x^n \ll e^x$

Similarly, $x^n \ll e^{\frac{x}{2}}$.

$$\Rightarrow \quad x^n e^{-x} \leq e^{\frac{x}{2}} \cdot e^{-x}$$

$$\Rightarrow \int_1^\infty x^n e^{-x}\, dx \leq \int_1^\infty e^{\frac{x}{2}} \cdot e^{-x}\, dx$$

$$= \int_1^\infty e^{-\frac{x}{2}}\, dx$$

$\therefore \int_1^\infty e^{-\frac{x}{2}}\, dx$ converges,

$\therefore \int_1^\infty x^n e^{-x}\, dx$ converges.